

application of position—which occurs in the Tuscan abacus, and in the Suampan of Inner Asia, has been twice independently invented, in the East under the Ptolemies, and in the West? or whether the system of position-value may not have been transferred by the direction of universal traffic from the Indian western peninsula to Alexandria, and subsequently have been given out amid the renewed dreams of the Pythagoreans as an invention of the founder of their sect? The bare possibility of ancient and wholly unknown combinations anterior to the sixtieth Olympiad is scarcely worthy of notice. Wherefore should a feeling of similar requirements not have severally given rise, among highly-gifted nations of different origin, to combinations of the same ideas?

While the algebra of the Arabs, by means of that which they had acquired from the Greeks and Indians, combined with the portions due to their own invention, acted so beneficially on the brilliant epoch of the Italian mathematicians of the Middle Ages, notwithstanding a great deficiency in symbolical designations, we likewise owe to the same people the merit of having furthered the use of the Indian numerical system from Bagdad to Cordova by their writings and their extended commercial relations. Both these effects—the simultaneous diffusion of the knowledge of the science of numbers and of numerical symbols with value by position—have variously, but powerfully, favored the advance of the mathematical portion of natural science, and facilitated access to the more abstruse departments of astronomy, optics, physical geography, and the theories of heat and magnetism, which, without such aids, would have remained unopened.

The question has often been asked, in the history of nations, what would have been the course of events if Carthage had conquered Rome and subdued the West? “We may ask with equal justice,” as Wilhelm von Humboldt* observes, “what would be the condition of our civilization at the present day if the Arabs had remained, as they long did, the sole possessors of scientific knowledge, and had spread themselves permanently over the West? A less favorable result would

sign to represent the descending negative scale for degrees and minutes both in his *Almagest* and in his *Geography*. The zero-sign was consequently in use in the West much earlier than the epoch of the invasion of the Arabs. (See my work above cited, and the memoir printed in *Crell's Mathematical Journal*, p. 215, 219, 223, and 227.)

* Wilhelm von Humboldt, *Ueber die Kawi-Sprache*, bd. i., s. cclxii. Compare, also, the excellent description of the Arabs in Herder's *Idcen zur Gesch. der Menschheit*, book xix., 4 and 5.