scribe an octahedron in the orbit of Venus; the circle inscribed in it will be Mercury's orbit. This is the reason of the number of the planets." The five kinds of polyhedral bodies here mentioned are the only "Regular Solids."

But though this part of the Mysterium Cosmographicum was a failure, the same researches continued to occupy Kepler's mind; and twenty-two years later led him to one of the important rules known to us as "Kepler's Laws;" namely, to the rule connecting the mean distances of the planets from the sun with the times of their revolutions. This rule is expressed in mathematical terms, by saying that the squares of the periodic times are in the same proportion as the cubes of the distances; and was of great importance to Newton in leading him to the law of the sun's attractive force. We may properly consider this discovery as the sequel of the train of thought already noticed. In the beginning of the Mysterium, Kepler had said, "In the year 1595, I brooded with the whole energy of my mind on the subject of the Copernican system. There were three things in particular of which I pertinaciously sought the causes why they are not other than they are; the number, the size, and the motion of the orbits." We have seen the nature of his attempt to account for the two first of these points. He had also made some essays to connect the motions of the planets with their distances, but with his success in this respect he was not himself completely satisfied. But in the fifth book of the Harmonice Mundi, published in 1619, he says, "What I prophesied two-andtwenty years ago as soon as I had discovered the Five Solids among the Heavenly Bodies; what I firmly believed before I had seen the Harmonics of Ptolemy; what I promised my friends in the title of this book (On the most perfect Harmony of the Celestial Motions), which I named before I was sure of my discovery; what sixteen years ago I regarded as a thing to be sought; that for which I joined Tycho Brahe, for which I settled in Prague, for which I have devoted the best part of my life to astronomical contemplations; at length I have brought to light, and have recognized its truth beyond my most sanguine expectations."

The rule thus referred to is stated in the third Chapter of this fifth Book. "It is," he says, "a most certain and exact thing that the proportion which exists between the periodic times of any two planets is precisely the sesquiplicate of the proportion of their mean distances; that is, of the radii of the orbits. Thus, the period of the earth is one year, that of Saturn thirty years; if any one trisect the proportion, that

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