Thus the satellites of Jupiter are not only disturbed by the sun, as the moon is, but also by each other, as the planets are. This mutual action gives rise to some very curious relations among their motions; which, like most of the other leading inequalities, were forced upon the notice of astronomers by observation before they were obtained by mathematical calculation. In Bradley's remarks upon his own Tables of Jupiter's Satellites, published among Halley's Tables, he observes that the places of the three interior satellites are affected by errors which recur in a cycle of 437 days, answering to the time in which they return to the same relative position with regard to each other, and to the axis of Jupiter's shadow. Wargentin, who had noticed the same circumstance without knowledge of what Bradley had done, applied it, with all diligence, to the purpose of improving the tables of the satellites in 1746. But, at a later period, Laplace established, by mathematical reasoning, the very curious theorem on which this cycle depends, which he calls the libration of Jupiter's satellites; and Delambre was then able to publish Tables of Jupiter's Satellites more accurate than those of Wargentin, which he did in 1789.24

The progress of physical astronomy from the time of Euler and Clairaut, has consisted of a series of calculations and comparisons of the most abstruse and recondite kind. The formation of Tables of the Planets and Satellites from the theory, required the solution of problems much more complex than the original case of the Problem of Three Bodies. The real motions of the planets and their orbits are rendered still further intricate by this, that all the lines and points to which we can refer them, are themselves in motion. The task of carrying order and law into this mass of apparent confusion, has required a long series of men of transcendent intellectual powers; and a perseverance and delicacy of observation, such as we have not the smallest example of in any other subject. It is impossible here to give any detailed account of these labors; but we may mention one instance of the complex considerations which enter into them. The nodes of Jupiter's fourth satellite do not go backwards,25 as the Newtonian theory seems to require; they advance upon Jupiter's orbit. But then, it is to be recollected that the theory requires the nodes to retrograde upon the orbit of the perturbing body, which is here the third satellite; and Lalande showed that, by the necessary relations of space, the latter motion may be retrograde though the former is direct.

24 Voiron, Hist. Ast. p. 822.

25 Bailly, iii. 175.