still these do not deprive Bernoulli of the merit of having pointed out the principle of Coexistent Vibrations, or divest that principle of its value in physical science.

Daniel Bernoulli's Memoir, of which we speak, was published at a period when the clouds which involve the general analytical treatment of the problem of vibrating strings, were thickening about Euler and D'Alembert, and darkening into a controversial hue; and as Bernoulli ventured to interpose his view, as a solution of these difficulties, which, in a mathematical sense, it is not, we can hardly be surprised that he met with a rebuff. The further prosecution of the different modes of vibration of the same body need not be here considered.

The sounds which are called *Grave Harmonics*, have no analogy with the Acute Harmonics above-mentioned; nor do they belong to this section; for in the case of Grave Harmonics, we have one sound from the co-operation of two strings, instead of several sounds from one string. These harmonics are, in fact, connected with beats, of which we have already spoken; the beats becoming so close as to produce a note of definite musical quality. The discovery of the Grave Harmonics is usually ascribed to Tartini, who mentions them in 1754; but they are first noticed[6] in the work of Sorge *On tuning Organs*, 1744. He there expresses this discovery in a query. "Whence comes it, that if we tune a fifth (2 : 3), a *third* sound is faintly heard, the octave below the lower of the two notes? Nature shows that with 2 : 3, she still requires the unity, to perfect the order 1, 2, 3." The truth is, that these numbers express the frequency of the vibrations, and thus there will be coincidences of the notes 2 and 3, which are of the frequency 1, and consequently give the octave below the sound 2. This is the explanation given by Lagrange,[7] and is indeed obvious.

CHAPTER V.

Problem of the Sounds of Pipes.

IT was taken for granted by those who reasoned on sounds, that the sounds of flutes, organ-pipes, and wind-instruments in general, con-

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[6] Chladni. *Acoust.* p. 264.

[7] *Mem. Tur.* i. p. 104.