the best we can attain to in our results.[1] An entirely new branch of investigation springs up—*viz.*, the theory of error, the doctrine of probability, and the investigation

and calculation were invented to deal practically with the problem. Up to 1781, when the new planet Uranus was discovered by Herschel, the interest centred mainly in the determination of the orbits of comets, which were assumed to be parabolic. Halley was the first to calculate these by means of tentative methods given by Newton in the 'Principia.' After 1781 the necessity arose of determining closed orbits, and a first attempt was made to do so by assuming circular orbits (neglecting the ellipticity) and neglecting the inclination of the plane of the orbit to that of the earth. But in the first year of this century neither the parabolic nor the circular figure of the orbits seemed to answer in the case of the new planet Ceres, nor could the inclination of the orbit be neglected. It required all the skill of Gauss to tackle the entire, unabbreviated problem, and this was done in his fundamental work 'Theoria motus corporum cœlestium.' As the 'Principia' form the foundation of all physical, so does the 'Theoria motus' of all calculating astronomy. A similar fundamental work which should take the next important step, solving generally the problem of the motion of a body which is attracted from more than one fixed or movable centre (the problem of three bodies), would mark the next great era in calculating astronomy. Hitherto this problem has only been treated under the assumption that the third attracting body disturbs the real orbit which has been calculated. The necessity of solving the problem of three bodies has made itself felt in the theory of the moon and other satellites, which stand under the influence of the main planet as well as the sun, and where therefore the ellipsis of Kepler cannot even be taken as a first approximation. And here again the necessity of taking into account the volume and the figures of the attracting bodies still further complicates the problem. On them depend the precession of the equinoxes and the irregularity of the precession known under the name of nutation.

[1] According to Wolf ('Handbuch der Astronomie,' vol. i. p. 128 *sqq.*) the merit of having first considered the best methods of dealing with errors of observation belongs to Picard (1670) and Roger Cotes ('Aestimatio errorum in mixta mathesi,' 1722). The former seems to have first used the apparently so obvious rule of taking the arithmetical mean of a number of observations, the latter introduced the notion of attributing to each observation its value or weight. Cotes accordingly found that the centre of gravity of a number of weighted points distributed over a plane coincided with the position of greatest probability. Gauss suspected that Tobias Mayer had already employed modern methods in his calculation of long series of observations, and he himself used what is termed after Legendre the "method of least squares" as early as 1795. It was not published till 1806 by Legendre, in his memoir 'Nouvelles méthodes pour la détermination des orbites des comètes.' Gauss published his methods in 1809 in the celebrated 'Theoria motus corporum cœlestium.' This method of finding the most probable result when a larger number of equations is given than unknown quantities