culation, to be accumulated. A real physical theory, however, was impossible until the notions suggested by common-sense were completely reversed, and an ideal construction put in the place of a seemingly obvious theory. This was done in astronomy at one stroke by Copernicus; in optics only gradually, tentatively, and hesitatingly. The purely geometrical rela-·tions of straight lines, which light seemed to resemble; of pencils of rays, which were bent back or altered in their direction at the surface of plane or curved mirrors and of transparent bodies; seemed to flow quite easily and naturally when in the seventeenth century the simple law of refraction had been added to that of reflexion, known already to the ancients. The sciences of catoptrics and dioptrics, with their application to the telescope and microscope, were thus so complete and useful that to many it must have seemed difficult and unnecessary to plunge into a new theory; ¹ especially

¹ It has always been the aim of "geometrical optics" to free itself from every hypothesis on the physical nature of light, and to deduce properties of light from a few simple geometrical constructions. Precisely in the same way all geometrical and many physical properties of the stellar system can be deduced from the kinematical formula of attraction, without discussing the nature of gravitation. This desideratum - so far as optics is concernedwas before the mind of Sir W. R. Hamilton, when, during the years 1824-33, he discovered and elaborated the theory of the "characteristic function, by the help of which all optical problems, whether on the corpuscular or on the undulatory theory, are solved by one common

process" (Tait, 'Light,' 2nd ed., p. 160). Owing to the difficulties which have more and more presented themselves in the fundamental conceptions of the wave-theory and the vibrating ether, of which we shall learn more in the sequel of this chapter, the desire to bring the phenomena of refraction under a purely geometrical formula, and to emancipate the optics of crystals from physical hypotheses, has become very pronounced. Huygens' geometrical construction of the ordinary and extraordinary rays in uniaxial crystals answered well. For biaxial crystals Fresnel had introduced the wave-surface, to which corresponds Hamilton's characteristic function. For didactic purposes, and for the practical applica-