very largely in all statistics, and vitiates them; and as regards coming events, our minds are in a state of expectation rather than of assurance. But events can be more or less probable, errors can be greater or smaller, cumulative or compensatory, and our expectations may be well- or ill-founded. And so there has arisen the science of Probabilities and of Chances, and the Theory of Error, two subjects intimately interwoven. former arose in the seventeenth century out of the frivolous or vicious practice of betting and gambling,1 whilst the latter was founded when astronomical observations accumulated, and the question presented itself how to combine them so as to arrive at the most reliable The greatest mathematicians and philosophers, result. such as Pascal, Huygens, and Leibniz, the Bernoullis, De Moivre, Laplace, Gauss and Poisson, have bestowed much thought on the subject,2 which has nevertheless been very differently judged-praised beyond measure by some, and ridiculed by others; sometimes pronounced to be merely common-sense put in figures, and then again wrapped up

13. The science of Chances.

¹ See supra, vol. i. p. 120 sqq.

contained in the seventh volume of the 'Jahresbericht der Deutschen Mathematiker Vereinigung' (Leipzig, 1899). The latter work is written on a different principle from that of Todhunter. Whereas Todhunter deals in separate chapters with the work of the foremost mathematicians on this subject, Prof. Czuber gives an independent historical and critical analysis of the different developments of the theory and its applications. Quite recently the same author has published an independent treatise on the subject (Leipzig, 1902).

In addition to the references given in vol. i., the following are of importance. The history of the Theory of Probabilities, as stated above, has been written by Isaac Todhunter. This history brings the subject down to the writings of Laplace, whose two works mentioned in the text still remain the two standard works on the science. In quite recent times the history has been written and brought up to date by Prof. Emanuel Czuber in his 'Entwickelung der Wahrscheinlichkeits-Theorie und ihre Anwendungen,'