elaborate calculations in astronomy, geodesy, and in various physical and statistical researches.

18. Method of Least Squares. Gauss.

Bound up with the theory of Error is the celebrated method of least Squares, first used by Gauss in 1795, published by Legendre in 1805 in his memoir 'On a New Method of Determining the Orbit of a Comet,' and elaborately discussed by Laplace, Gauss, and many subsequent writers to this day.[1] It may be looked upon as an extension or generalisation of the common-sense

[1] In addition to the references given in the notes to pp. 120 and 183 of vol. i., I can now recommend two excellent summary accounts of the history and theory of the method of least squares—the one in Prof. Czuber's 'Bericht,' quoted above (pp. 150 to 224); the other in Prof. Edgeworth's article on "The Law of Error" in the Supplement to the last edition of the 'Ency. Brit.' (vol. xxviii., 1902, p. 280, &c.) Prof. Cleveland Abbe, in a "historical note on the method of least squares" ('American Journal of Mathematics,' 1871), has drawn attention to the fact, that already in 1808 Prof. R. Adrain of New Brunswick had arrived at an expression for the law of error identical with the formula now generally accepted, without knowing of Gauss's and Legendre's researches. See a paper by Prof. Glaisher in the 39th vol., p. 75, of the 'Transactions of the Royal Astronomical Society.' The logical and mathematical assumptions upon which the method is based have been submitted to repeated and very searching criticisms, many rigid proofs having been attempted, and every subsequent writer having, seemingly, succeeded in discovering flaws in the logic of his predecessors. In connection with another subject, I may have occasion to point out how nearly all complicated logical arguments have shown similar weakness, and how, in many cases, the conviction of the correctness or usefulness of the argument comes back to the self-evidence of some common-sense assumption, which cannot be proved, though it may be universally accepted. Many analysts have tried to prove the correctness of the everyday process of taking the arithmetical mean, but have failed. Prof. Czuber says, *inter alia* (*loc. cit.*, p. 159): "The fact that Gauss, in his first demonstration of the method of least squares, conceded to the arithmetical mean a definite theoretical value, has been the occasion for a long series of investigations concerning the subject, which frequently showed the great acumen of their authors. The purpose aimed at—viz., to show that the arithmetical mean is the only result which ought to be selected as possessing cogent necessity, hereby giving a firm support to the intended proofs, has not been attained, because it cannot be attained. Nevertheless, these investigations have their worth because they afford clear insight into the nature of all average values and into the position which the arithmetical average occupies among them."