such moving crowds turn out to be exactly those laws which Boyle, Charles, Gay-Lussac, Dalton, and Avogadro had found out by direct experiments with gaseous bodies. James Clerk - Maxwell was the first to recognise the great importance of the statistical methods, and to apply them in an exhaustive manner.<sup>1</sup> And we witness here the same spectacle which presented itself in the history of the theory of probabilities. Problems which are to be solved by the mere application of a few rules dictated by common-sense and an exercise of common logic, present in their complexity such a multitude of traps, snares, and pitfalls, that it required the successive application of the highest intellects to free the reasoning from insidious errors, and put the results on

<sup>1</sup> The manner in which Joule dealt with the problem of a large crowd of moving particles in his memoir of 1851 was not strictly statistical, inasmuch as he dealt with an average velocity of the molecules, and assumed that all the molecules of a gas moved with the same velocity. Clausius, in his memoir of 1857, made use of assumptions which were more in conformity with nature: he had, accordingly, to employ the calculus of probabilities. Clerk - Maxwell's occupation with the subject dates from the year 1859, when he read his paper, "Illustrations of the Dynamic Theory of Gases," Part I. (published in the 'Phil. Mag.,' 4th series, vol. xix. p. 19, reprinted in 'Scientific Papers,' vol. i.) He showed that "the velocities are distributed among the particles according to the same law as errors are distributed among the observations in the theory of the method of least squares. The velocities range from 0 to  $\infty$ , but the number

of those having great velocities is comparatively small." If we leave out Joule's imperfect attempt to employ the statistical method, one of the first applications of the method of averages to a physical problem is to be found in Sir G. G. Stokes's paper "On the Composition of Streams of Polarised Light from different Sources" ('Camb. Phil. Trans.,' 1853), where he shows "what will be the average effect of a very great number of special sources of light: thus giving one of the earliest illustrations of the use, in physics, of the statistical methods of probabilities. . . . From this point of view the uniformity of optical phenomena becomes quite analogous to the statistical species of uniformity, which is now found to account for the behaviour of the practically infinite group of particles forming a cubic inch of gas" (P. G. Tait, 'Light,' 2nd ed., 1889, p. 237).

28. Mathematical representation of experimental laws.