

for they mark more the attitude of the beholder than the things which he regards. It is true that a very small addition to our actual knowledge of facts, like the sudden appearance of some characteristic feature in a landscape, may sometimes entirely alter the whole aspect, induce us to abandon our accustomed views, and call up suddenly an unforeseen train of ideas; in such a case, perhaps, this insignificant discovery becomes historically interesting, although it is mainly by the altered trains of thought which it has evoked that it has become important to us.

2.
Difference
between
thought and
knowledge.

The difference of scientific knowledge and scientific thought is thus owing to the two factors which are involved—the facts of science or nature on the one side and the scientifically thinking mind on the other. Now it might appear as if this difference vanished when we approach the abstract science of mathematics, or at least that of number; for in numbering and counting we have really only to do with a process of thought, and it would seem as if the science of number were itself the science of thought, or at least a portion of it. In fact, the question arises, Is there any difference between mathematical science and mathematical thought? Some considerations might induce us to think that there is not. On the other side, I shall try to show in this chapter that there is, and that the development of mathematics during our period has brought this out very clearly and prominently.

3.
Popular
prejudices
regarding
mathe-
matics.

There is an opinion current among many thinking persons who have not occupied themselves with mathematical science, though they may be very efficient in