ceases as soon as discussions arise which cannot benefit those who use the instrument for the purposes of application in mechanics, astronomy, physics, statistics, and other sciences. At the other extreme we have those who are animated exclusively by the love of pure science. To them pure mathematics, with the theory of numbers [1] at the head, is the one real and genuine science, and the applications have only an interest in so far as they contain or suggest problems in pure mathematics. They are mainly occupied with examining and strengthening the foundations of mathematical reasoning and purifying its methods, inventing rigorous proofs, and testing the validity and range of applicability of current conceptions. We may say that the former are led by practical, the latter by philosophical, interests, and these latter may be either logical or ontological,[2]

102); the latter was energetically repudiated by Sylvester in his famous Address to the first section of the British Assoc. at Exeter (1869, 'Report,' &c., p. 1, &c.)

[1] Gauss considered mathematics to be "the Queen of the Sciences, and arithmetic the Queen of Mathematics. She frequently condescends to do service for astronomy and other natural sciences, but to her belongs, under all circumstances, the foremost place" (see 'Gauss zum Gedächtniss,' by Sartorius von Waltershausen, Leipzig, 1856, p. 79). Cayley's presidential Address to the British Association, 1883, has been frequently quoted: "Mathematics connect themselves on one side with common life and the physical sciences; on the other side with philosophy in regard to our notions of space and time and the questions which have arisen as to the universality and necessity of the truths of mathematics, and the foundation of our knowledge of them. I would remark here that the connection (if it exists) of arithmetic and algebra with the notion of time is far less obvious than that of geometry with the notion of space" ('Mathematical Papers,' vol. xi. p. 130). In addition to founding higher arithmetic, Gauss occupied himself with the foundations of geometry, and, as he expected much from the development of the theory of numbers, so he placed "great hopes on the cultivation of the *geometria situs*, in which he saw large undeveloped tracts which could not be conquered by the existing calculus" (Sartorius, *loc. cit.*, p. 88).

[2] To this might be added the psychological interest which attaches to mathematical conceptions. The late Prof. Paul Du Bois - Reymond occupied himself