5. Twofold interest in mathematics.

inasmuch as number and form are considered to be the highest categories of human thought, or likewise as the ultimate elements of all reality. These two interests existed already in antiquity,[1] as the word "geometry"

much with the question. See the following works: 'Die Allgemeine Functionentheorie,' part i., Tübingen, 1882; 'Ueber die Grundlagen der Erkenntniss in den exacten Wissenschaften,' Tübingen, 1890; and his paper "Ueber die Paradoxien des Infinitärcalcüls" ('Mathematische Annalen,' vol. ix. p. 149). In addition to the two main interests which attach to mathematical research, and which I distinguish as the practical and the philosophical, a third point of view has sprung up in modern times which can be called the purely logical. It proposes to treat any special development of mathematical research with the aid of a definite, logically connected complex of ideas, and not to be satisfied to solve definite problems with the help of any methods which may casually present themselves, however ingenious they may be. In this way the great geometrician, Jacob Steiner, *e.g.*, refused the assistance of analysis in the solution of geometrical problems, conceiving geometry as a complete organism which should solve its problems by its own means. This view has been much strengthened by the development in modern times of the theory of Groups; a group of operations being defined as a sequence of such operations as always lead back again to operations of the same kind. Mathematical rigorists in this sense would look upon the use of mixed methods or operations not belonging to the same group with that kind of disfavour with which we should regard an essayist who could not express his ideas in pure English, but was obliged to import foreign words and expressions. It is interesting to see that the country which has offended most by the importation of foreign words — namely, Germany—is that in which this purism in mathematical taste has found the most definite expression. (See, *inter alia*, Prof. Friedrich Engel's Inaugural Lecture, "Der Geschmack in der neueren Mathematik," Leipzig, 1890, as also Prof. F. Klein's suggestive tract, 'Vergleichende Betrachtungen über neuere Geometrische Forschungen,' Erlangen, 1872.)

[1] The literature of this subject is considerable. I confine myself to two works. The late eminent mathematician, Hermann Hankel, of whom more in the sequel of this chapter, besides showing much originality in the higher branches of the science, took great interest in its philosophical foundations and historical beginnings. In 1870 he published a small but highly interesting volume, 'Zur Geschichte der Mathematik in Alterthum und Mittelalter' (Leipzig, Teubner). We have, besides, the great work of Prof. Moritz Cantor, 'Vorlesungen über Geschichte der Mathematik,' in three large volumes (Leipzig, Teubner). It brings the history down to 1758. Referring to the two interests which led to mathematical investigations, Hankel says (p. 88): "From the moment that Greek philosophers begin to attract our attention through their mathematical researches, the aspect which mathematics present