our century, among whom I only mention Gauss, Cauchy, and Weierstrass, attempted to do for the new science which was created during the two preceding centuries. As Prof. Klein says, "We are living in a critical period, similar to that of Euclid."[1]

[1] See 'The Evanston Colloquium, Lectures on Mathematics delivered in August and September 1893,' by Felix Klein, notably Lecture vi. In this lecture Prof. Klein explains his view (to which he had given utterance in his address before the Congress of Mathematics at Chicago: 'Papers published by the American Mathematical Society,' vol. i. p. 133. New York, 1896) on the relation of pure mathematics to applied science. This view is based upon the distinction between what he calls the "naïve and the refined intuition." . . . "It is the latter that we find in Euclid; he carefully develops his system on the basis of well-formulated axioms, is fully conscious of the necessity of exact proofs, clearly distinguishes between the commensurable and the incommensurable, and so forth. . . . The naïve intuition, on the other hand, was especially active during the period of the genesis of the differential and integral calculus. Thus we see that Newton assumes without hesitation the existence, in every case, of a velocity in a moving point, without troubling himself with the inquiry whether there might not be continuous functions having no derivative."

In the opinion of Prof. Klein "the root of the matter lies in the fact that the naïve intuition is not exact, while the refined intuition is not properly intuition at all, but arises through the logical development from axioms considered as perfectly exact."

In the sequel Prof. Klein shows that the naïve intuition imports into the elementary conceptions elements which are left out in the purely logical development, and that this again leads to conclusions which are not capable of being verified by intuition, no mental image being possible. Thus, for instance, the abstract geometry of Lobatchevsky and Riemann led Beltrami to the logical conception of the pseudo-sphere of which we cannot form any mental image. Similar views to those of Prof. Klein have been latterly expressed by H. Poincaré in his suggestive volume 'La Science et l'Hypothèse' (Paris, 1893). He there says (p. 90): ". . . L'expérience joue un rôle indispensable dans la genèse de la géométrie; mais ce serait une erreur d'en conclure que la géométrie est une science expérimentale, même en partie. . . . La géométrie ne serait que l'étude des mouvements des solides; mais elle ne s'occupe pas en réalité des solides naturels, elle a pour objet certains solides idéaux, absolument invariables, qui n'en sont qu'une image simplifiée et bien lointaine. . . . Ce qui est l'objet de la géométrie c'est l'étude d'un 'groupe' particulier; mais le concept général de groupe préexiste dans notre esprit au moins en puissance. . . . Seulement, parmi tous les groupes possibles, il faut choisir celui qui sera pour ainsi dire l'étalon auquel nous rapporterons les phénomènes naturels." This distinction between the mathematics of intuition and the mathematics of logic has also been forced upon us from quite a different quarter. The complica-