http://geology.19thcenturyscience.org/books/1906-Merz-HistEurThot/README.htm

this second and more hidden line of research the logical side of the new development. It corresponds to the work which Euclid performed in ancient geometry, the framing of clear definitions and of unambiguous axioms; proceeding from these by rigorous reasoning to the theorems of the new science.[1] But the translation of geometrical and mechanical conceptions into those of generalised arithmetic or algebra brought with it a logical problem of quite a novel kind which has given to modern mathematics quite a new aspect. This new problem is the retranslation of algebraical—*i.e.*, of general—formulæ into geometrical conceptions—the geometrical construction of algebraical expressions. It is the inverse operation of the former. In this inversion of any given operation lies the soul and principle of all mathematical progress, both in theory and in application.[2] The invention of

10. Inverse operations.

[1] Referring specially to the definition of a "function" or mathematical dependence, a conception introduced by Euler, but not rigorously defined by him, M. Poincaré says, *loc. cit.*: "Au commencement du siècle, l'idée de fonction était une notion à la fois trop restreinte et trop vague. . . . Cette définition, il fallait la donner: car l'analyse ne pouvait qu'à ce prix acquérir la parfaite rigueur." In its generality this task was performed in the last third of the century by Weierstrass, but the necessity of this criticism of the formulæ invented by modern mathematics dates from the appearance of Cauchy's 'Mémoire sur la théorie des intégrales définies' of 1814, which Legendre reported on in this sense, but which was not published till 1825.

[2] The operations referred to are generally of two kinds: first, there is the operation of translating geometrical relations, intuitively given, into algebraical relations; and, secondly, the operation of extending algebraical relations by going forward or backward in the order of numbers, usually given by indices. In each case the new relations arrived at require to be interpreted, and this interpretation leads nearly always to an extension of knowledge or to novel conceptions. A simple example of the first kind presents itself in the geometrical construction of the higher powers of quantity. Having agreed to define by a the length of a line, by a^2 an area, what is the meaning of a^3 a^4 . . . a^n? Can any geometrical meaning be attached to these symbols? An example of the