the seventeenth century afforded two grand occasions for such progress, and the creation through it of novel mathematical ideas. The translation of geometrical con-

second class is the following: having defined the symbols

$$\frac{dy}{dx}, \frac{d^2y}{dx^2} \cdots \frac{d^ny}{dx^n},$$

an operation suggests itself in the inverse order, the indices or their reciprocals (inversions) being taken negatively. Can any meaning be attached to these latter symbols? Further, if the operation denoted by going on from one of these symbols to the next is known and feasible, how can the inverse operation be carried out? In the first class of problems we proceed from an intuitively given order to a purely logical order, and have in the sequel to go back from the purely logical order to an intuitive order of ideas. In the second case, having followed a certain logical order, we desire to know what the inversion of this order will produce and how it can be carried out. The view that the direct and indirect processes of thought form the basis of all mathematical reasoning, and an alternation of the two the principle of progress, has been for the first time consistently expounded by Hermann Hankel in his 'Theorie der Complexen Zahlen-Systeme,' Leipzig, 1867. But it had already been insisted on by George Peacock in his "Report," &c., contained in the 3rd vol. of the 'Reports of the Brit. Assoc.,' 1833, where he says (p. 223): "There are two distinct processes in Algebra, the direct and the inverse, presenting generally very different degrees of difficulty. In the first case, we proceed from defined operations, and by various processes of demonstrative reasoning we arrive at results which are general in form though particular in value, and which are subsequently generalised in value likewise; in the second, we commence from the general result, and we are either required to discover from its form and composition some equivalent result, or, if defined operations have produced it, to discover the primitive quantity from which those operations have commenced. Of all these processes we have already given examples, and nearly the whole business of analysis will consist in their discussion and development, under the infinitely varied forms in which they will present themselves."

It is extraordinary how little influence this very interesting, comprehensive, and up-to-date report on Continental mathematics, including the works of Gauss, Cauchy, and Abel, seems to have had on the development of English mathematics. But the latter have through an independent movement—viz., the invention of the Calculus of Operations—led on to the radical change which has taken place in recent mathematical thought. This change, which can be explained by saying that the science of Magnitude must be preceded by the doctrine of Forms or Relations, and that the science of Magnitude is only a special application of the science of Forms, was independently prepared by Hermann Grassman, of whom Hankel says (*loc. cit.*, p. 16): "The idea of a doctrine of Forms which should precede a doctrine of Magnitude, and of considering the latter from the point of view of the former, . . . remained of little value for the development