ceptions into algebraical language suggested the inverse operation of interpreting algebraical terms by geometrical conceptions, and led to an enormous extension of geometrical knowledge.[1] Further, the infinitesimal methods through which curves and curved surfaces were conceived as being made up of an infinite number of infinitesimally small, straight—*i.e.*, measurable—lines, led to the inverse problem; given any algebraical operations which obtain only in infinitesimally small dimensions—*i.e.*, at the limit—how do they sum up to finite quantities and

of mathematics, so long as it was only used to prove theorems which besides being already known, were sufficiently though merely empirically proved. It was H. Grassmann who took up this idea for the first time in a truly philosophical spirit and treated it from a comprehensive point of view." Hankel also refers to Peacock as well as to De Morgan, whose writings, however, he was insufficiently acquainted with (ibid., p. 15). In quite recent times Mr A. N. Whitehead has conceived "mathematics in the widest signification to be the development of all types of formal, necessary, deductive reasoning," and has given a first instalment of this development in his 'Treatise on Universal Algebra' (vol. i., Cambridge, 1898). See the preface to this work (pp. 6, 7).

[1] A good example of the use of the alternating employment of the intuitive (inductive) and the logical (deductive) methods is to be found in the modern doctrine of curves. The invention of Descartes, by which a curve was represented by an equation, led to the introduction of the conception of the "degree" or "order" of a curve and its geometrical equivalent; whereas the geometrical conception of the tangent to a curve led to the distinction of curves according to their "class," which was not immediately evident from the equation of the curve but which led to other analytical methods of representation where the tangential properties of curves became more evident. A third method of studying curves was introduced by Plücker (1832), who started from "the singularities" which curves present, defined them, and established his well-known equations. A further study of these "singularities" led to the notion of the "genus" or "deficiency" (Cayley) of a curve. The gradual development of these and further ideas relating to curves is concisely given in an article by Cayley on "Curve" in the 6th vol. of the 'Encyclopædia Britannica,' reprinted in Cayley's collected papers, vol. xi. This article furnishes also a good example of the historical treatment of a purely mathematical subject by showing, not so much the progress of mathematical knowledge of special things, as the development of the manner in which such things are looked at—*i.e.*, of mathematical thought.