

of this kind was given by Brook Taylor, and somewhat modified by Maclaurin. It embraced all then known and many new series, and was employed without hesitation by Euler and other great analysts. In the beginning of the century, Poisson, Gauss, and Abel drew attention to the necessity of investigating systematically what is termed the convergency¹ of a series. As a specimen of this kind of research, Gauss published, in 1812, an investigation of a series of very great generality and importance.² We can say that through these two isolated memoirs of Gauss, the first of the three on equations, published in 1799, and the memoir on the series of 1812, a new and more rigorous treatment of the infinite and the continuous as mathematical conceptions was introduced into analysis, and that in both he showed the necessity of extending the system of numbering and measuring by the conception of the complex quantity. But it cannot be maintained that Gauss succeeded in impressing the new line of thought upon the science of

¹ A very good account of the gradual evolution of the idea of the convergency of a series will be found in Dr R. Reiff's 'Geschichte der unendlichen Reihen' (Tübingen, 1899, p. 118, &c.) Also in the preface to Joseph Bertrand's 'Traité de Calcul Différentiel' (Paris, 1864, p. xxix, &c.) According to the latter Leibniz seems to have been the first to demand definite rules for the convergency of Infinite Series, for he wrote to Hermann in 1705 as follows: "Je ne demande pas que l'on trouve la valeur d'une série quelconque sous forme finie; un tel problème surpasserait les forces des géomètres. Je voudrais seulement que l'on trouvât moyen de

décider si la valeur exprimée par une série est possible, c'est-à-dire convergente, et cela sans connaître l'origine de la série. Il est nécessaire, pour qu'une série indéfinie représente une quantité finie, que l'on puisse démontrer sa convergence, et que l'on s'assure qu'en la prolongeant suffisamment l'erreur devient aussi petite que l'on veut." In spite of this, Leibniz, through his treatment of the series of Grandi, $1 - 1 + 1 - 1$, &c., the sum of which he declared to be $\frac{1}{2}$, seems to have exerted a baneful influence on his successors, including Euler (see Reiff, *loc. cit.*, pp. 118, 158).

² The memoir on the Hypergeometrical series.