France, somewhat later also in England and Germany. In the latter country, the highly original writings of Abel, and the independent labours of Jacobi, opened out an entirely new branch of higher mathematics, beginning with the discovery of the property of double periodicity of certain functions.[1] This extensive and fruitful province of analysis for a time retarded the revision and extension of the groundwork of mathematical reasoning which Cauchy had begun, and upon which Gauss evidently desired to make the extension of higher mathematics proceed.[2]

[1] Before the discovery of the functions with a double period, functions with one period were known: the circular and exponential functions—the former possessing a real, the latter an imaginary, period. The elliptic functions turned out to "share simultaneously the properties of the circular functions and exponential functions, and whilst the former were periodical only for real, the latter only for imaginary, values of the argument, the elliptic functions possessed both kinds of periodicity." This great step became clear when it occurred to Abel and Jacobi independently to form functions by inversion of Legendre's elliptic integral of the first kind. The two fundamental principles involved in this new departure were thus the process of inversion and the use of the imaginary, as a necessary complement to the real, scale of numbers. The share which belongs independently to Abel and Jacobi has been clearly determined since the publication of the correspondence of Jacobi with Legendre during the years 1827-32 (reprinted in Jacobi's 'Gesammelte Werke,' ed. Borchardt, vol. i., Berlin, 1881), and of the complete documents referring to Abel, which are now accessible in the memorial volume published in 1902. A very lucid account is contained in a pamphlet by Prof. Königsberger, entitled 'Zur Geschichte der Theorie der Elliptischen Transcendenten in den Jahren 1826-29' (Leipzig, 1879).

[2] Of the four great mathematicians who for sixty years did the principal work in connection with elliptic functions — viz., Legendre (1752-1833), Gauss (1777-1855), Abel (1802-29), and Jacobi (1804-51), each occupied an independent position with regard to the subject, —suggested originally by Euler, and important for the practical applications which it promised. Legendre during forty years, from 1786 onward, worked almost alone: he brought the theory of elliptic integrals, which had occurred originally in connection with the computation of an arc of the ellipse, into a system, and to a point beyond which the then existing methods seemed to promise no further advance. This advance was, however, secured by the labours of Jacobi through the introduction of the novel principles referred to in the last note. Two years before Jacobi's publication commenced, Abel had already approached the subject from an entirely different and much more