

That such a revision had become necessary was seen, slowly if in many quarters, but it did not become generally recognised till late in the century, when thinkers of

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general point of view. "Abel," as Monsieur L. Sylow says ('Mémorial des études d'Abel,' p. 14), "était avant tout algébriste. Il a dit lui-même que la théorie des équations était son sujet favori, ce qui d'ailleurs apparaît clairement dans ses œuvres. Dans ses travaux sur les fonctions elliptiques, le traitement des diverses équations algébriques dont cette théorie abonde est mis fortement en évidence, et dans le premier de ces travaux, la résolution de ces équations est même indiquée comme étant le sujet principal. Qui plus est, la théorie des équations était entre ses mains l'instrument le plus efficace. Ce fut ainsi sans aucun doute la résolution de l'équation de division des fonctions elliptiques qui tout d'abord le conduisit à la théorie de la transformation. Elle joue encore un rôle capitale dans sa démonstration du théorème dit théorème d'Abel, et dans les recherches générales sur les intégrales des différentielles algébriques qui se trouvent dans son dernier mémoire le 'Précis d'une Théorie des fonctions elliptiques.'" But whilst Abel certainly took a much more general view than either Legendre or Jacobi, both of whom came to a kind of deadlock on the roads they had chosen (Jacobi, when he attempted to extend the theory of the periodicity of functions), it is now quite clear that Gauss viewed the whole subject almost thirty years before Abel and Jacobi entered the field from a still more general point of view. Already, in 1798, when he was only twenty-one, he must have recognised the necessity of enlarging and defining the fundamental conceptions of algebra and of functionality or mathematical dependence; and it is very likely that the magnitude of the

undertaking, for which his astronomical labours left him no time, debarred him from publishing the important results which he had already attained, and which covered to a great extent the field cultivated in the meantime by Abel and Jacobi, leaving only the celebrated theorem of the former (referring to the algebraical comparison of the higher non-algebraical functions) and the discovery of a new function on the part of Jacobi (his Theta function) as the two great additions which we owe to them in this line of research (see Königsberger, *loc. cit.*, p. 104). In this recognition of the fundamental change which mathematical science demanded, and its bearing upon these special problems here referred to, Gauss must have for a long time stood alone; for his great rival Cauchy, to whom we are mainly indebted for taking the first steps in this direction, did not for many years apply his fundamental and novel ideas to the theory of elliptic functions, which up to the year 1844, when Hermite entered the field, were almost exclusively cultivated by German and Scandinavian writers (see R. L. Ellis, "Report on the recent Progress of Analysis," Brit. Assoc., 1846; reprinted in 'Mathematical and other Writings,' p. 311). Nor could it otherwise be explained how Cauchy could keep the manuscript of Abel's great memoir without ever occupying himself with it, and thus delay its publication for fifteen years after it had been presented to the Academy. (See the above-mentioned correspondence between Legendre and Jacobi, 1829; also Sylow, p. 31.)