degree were arrived at. The forward or direct process was easy enough, though even here assumptions or arbitrary rules were included which escaped notice for a long time; but the real labour of the analysts only began with the inverse problem—viz., given any compound quantity, similar in structure to those directly produced by multiplication of binomials, to find the factors or binomials out of which it can be compounded. Now it was found that as in the arithmetical process of division, the invention of fractional quantities; as in that of extraction of roots, the irrational quantities had to be introduced: so in the analysis of compound algebraical expressions into binomial factors, a new quantity or algebraical conception presented itself. It was easily seen that this analysis could be carried out in every case only by the introduction of a new unit, algebraically expressed by the square root of the negative unity. There was no difficulty in algebraically indicating the new quantity as we indicate fractions and irrational quantities; the difficulty lay in its interpretation as a number. Since the time of Descartes geometrical representations of algebraical formulæ had become the custom, and it was therefore natural when once the new, or so-called imaginary, unit was formally admitted, that a geometrical meaning should be attached to it.

19. The geometrical and the logical problems.

Out of the scattered beginnings of these researches two definite problems gradually crystallised: the one, a purely formal or mechanical one — viz., the geometrical representation of the extended conception of quantity, of the complex quantity; the other, a logical