

or philosophical one—viz., the clearer definition of the assumptions or principles which underlie arithmetical and algebraical reasoning. And if algebraical, then also geometrical reasoning. Both problems seem to have presented themselves to the youthful mind of Gauss, as is evident from his correspondence with Bessel¹ and Schumacher, and from his direct influence on Bolyai,² Möbius, and Von Staudt, perhaps also indirectly on Lobatchevsky.³ It does not, however, appear as if he

¹ See especially the letters of Gauss to Bessel, dated November and December 1811 and May 1812 ('Briefwechsel,' Leipzig, 1880, p. 151 sqq.)

² Bolyai, the elder (1775-1856), was a student friend of Gauss in the years 1797 to 1799, and kept up a correspondence with him during half a century. This correspondence has now been published by F. Schmidt and P. Stäckel, Leipzig, 1899, with a supplement containing some information about this extraordinary man. His son, Johann Bolyai (1802-60), is the author of the celebrated "Appendix, scientiam spatii absolute veram exhibens," which was attached to his father's 'Tentamen, juventutem . . . in elementa matheseos puræ . . . introducendi,' 1832. The tract seems to have been written in 1823. A translation, with introduction, has been published by Dr G. Bruce Halsted ('Neomonic Series,' vol. iii. 4th ed., Austin, Texas, 1896). When the elder Bolyai sent to Gauss in the year 1831 to 1832 a copy of his son's tract and of his own work on Geometry, Gauss expressed great surprise at the contents of the former. (See his letter of March 6, 1832.) His remarks that the younger Bolyai had anticipated some of his own ideas on the

subject, remind one of a similar remark which he made, May 30, 1828, to Schumacher with reference to Abel's "Memoir on Elliptic Functions" in vol. ii. of Crelle's 'Journal' (see Gauss, 'Werke,' vol. iii. p. 495). In both cases he felt himself relieved from the necessity of publishing his own results, though, so far as those referring to the foundations of geometry are concerned, it does not appear that his ideas had arrived at that state of maturity which the publication of his posthumous papers has proved to have been attained in his treatment of the higher functions. Indeed little or nothing of prime importance has been found among his papers referring to the principles of geometry; and he stated to Bolyai that though he had intended to commit his views to paper, so that they should not be lost, he had not intended to publish anything during his lifetime.

³ It is doubtful whether Gauss's speculations had any influence on the younger Bolyai's theory, and still more so as regards Lobatchevsky, whose first tract appeared in the 'Kazan Messenger,' 1829 to 1830, but dates back probably to 1826. Inasmuch, however, as the younger Bolyai must have become acquainted