

had arrived at any finality in his speculations, and, beyond occasional hints which have only subsequently become intelligible, the love of finish exhibited in all his published writings prevented him from giving to the world the suggestive ideas which evidently formed the groundwork of his mathematical labours. There is no doubt that—like Goethe in a very different sphere—Gauss anticipated individually the developments in the sphere of mathematical thought down to the end of the century. The interpretation of the complex quantity had been given by Wessel, Buée, and Argand¹ in the early years of the century; but it remained unnoticed till it received the sanction of Gauss in a celebrated memoir referring to the theory of numbers, and until in

through his father with the speculations of the youthful Gauss, and as Lobatchevsky was a pupil of another student friend of Gauss in the person of Prof. Bartels, it is not unlikely that the interest which these thinkers took in the subject can be originally traced to the same source. (See Dr Halsted's address on Lobatchevsky, 'Neomonic Series,' vol. i., 1894.) A complete bibliography of the earlier papers, referring to the so-called "non-Euclidean" literature down to 1878, is given by Dr Halsted in the first two vols. of the 'American Journal of Mathematics': the most recent publications are those of the Hon. B. A. W. Russell in his work, 'The Foundations of Geometry' (1897) and his excellent article on "Non-Euclidean Geometry" in the 28th vol. of the 'Ency. Brit.' See also Klein's lithographed lectures on 'Nicht-Euklidische Geometrie,' Göttingen, 1893.

¹ The first somewhat exhaustive historical statement as to the

geometrical representation of the complex or imaginary quantity was given by Hankel in the above-mentioned work (see above, note, p. 645), p. 82. He there says, after discussing the claims of others,—notably of Gauss,—that Argand in his 'Essai' of the year 1806 (re-edited by Hoüel, 1874) "had so fully treated of the whole theory that later nothing essentially new was added, and that, except a publication of still earlier date were found, Argand must be considered the true founder of the representation of complex quantities in the plane." Such an earlier publication has indeed been met with in a tract by Caspar Wessel, which was presented to the Danish Academy in 1797, and published in 1799. Having been overlooked, like Argand's 'Essai,' it has now been republished at Copenhagen, 1897, with the title 'Essai sur la représentation de la direction' (see 'Encyk. Math. Wissenschaften,' vol. i. p. 155).