this country the labours of De Morgan and of Sir William Rowan Hamilton gave the matter a further and very important extension.[1] It was also in this country that the second problem, the critical examination of the principles which underlie the process of legitimate generalisation of algebra, received distinct attention. To George Peacock, and to the school of algebraists which followed him, is due the merit of having brought out clearly the three fundamental laws of symbolical reasoning now generally admitted in text-books on the subject—the associative, distributive, and commutative principles. That these principles were to a great extent conventional, or empirically adopted from ordinary arithmetic, and in consequence not necessarily indispensable for a consistent system of symbolical reasoning, has been generally admitted ever since Sir William Rowan Hamilton, after ten years of labour, succeeded in establishing a new calculus—the method of quaternions, in which the commutative principle of multiplication is dropped. This

20. Quaternions.

[1] Far more important than the suggestions or artifices mentioned in the foregoing note, and which since the time of Argand and Gauss have been variously modified, is the conception that our common numbers do not form a complete system without the addition of the imaginary unit, but that with the introduction of a second unit "numbers form a universe complete in itself, such that, starting in it, we are never led out of it. There may very well be, and perhaps are, numbers in a more general sense of the term; but in order to have to do with such numbers (if any) we must start with them" (Cayley in art. "Equation," 'Ency. Brit.'; 'Coll. Works,' vol. xi. p. 503). There seems little doubt that this conception was first clearly established in the mind of Gauss, and that none of the contemporary writers can be shown to have had a similarly clear insight. Since this has become generally recognised—and we owe this recognition probably to the independent labours of Grassmann and Riemann—the discussion of the whole subject has been raised to a much higher level, as may be seen by comparing the Report of Peacock, quoted above, with the discussion of Hankel (*loc. cit.*), and still more with the exhaustive article by Prof. E. Study in vol. i., 'Encyk. Math. Wiss.,' pp. 147-184.