calculus was shown to be of special use in expressing the relations of spherical trigonometry. Two terms expressing definite notions special to geometry, by which science has been enriched and practical application greatly simplified, are an outcome of this line of research. These are the terms "vector," to express the notion of directed magnitude—*i.e.*, of direction and magnitude combined as distinguished from magnitude and position alone; and the notion of an "operator" which changes direction and magnitude as an ordinary multiplier changes magnitude only.[1] It was shown by Argand and others that the

[1] These two notions, which have their origin in the writings of Hamilton on the one side and the Calculus of Operations on the other, belong to this country and to a period during which mathematical researches were carried on in a fragmentary manner, and much out of contact with the contemporary mathematics of the Continent. Both the Calculus of Quaternions of Hamilton and the Calculus of Operations were looked upon for a long time as curiosities (as was also the Barycentric Calculus of Möbius in Germany). Gradually, however, the valuable ideas which were contained in them became recognised as much from the practical as from the theoretical point of view. In the former interest the application of Vector Analysis or the Algebra of Directed Quantities received a great impetus when the need was felt of having an algebra of "physical quantities." This found expression in the writings of Clerk-Maxwell. (See his 'Treatise on Electricity and Magnetism,' vol. i. p. 8, 2nd ed., as also his paper on "The Mathematical Classification of Physical Quantities," 1871. 'Coll. Papers,' vol. ii. p. 257.) In the practical application of electrical theories these notions have since become indispensable, and the subject has received increasing attention, notably in America, which holds a foremost place in the development of electrical science and its application. Mathematicians of the first order, such as J. Willard Gibbs, have published text-books on the subject, whilst other electricians of eminence, such as Mr Oliver Heaviside, have elaborated special forms of the Directional Calculus to serve their purposes. In Dynamics the Dublin School, represented after the death of Hamilton by Sir Robert S. Ball (in his 'Theory of Screws,' 1876), has had an important influence in the introduction of novel and more appropriate methods which have gradually permeated the general treatment of the subject. Whilst there is no doubt that for a long time the Calculus of Quaternions was the only methodical elaboration of these novel and useful ideas, it was overlooked that simultaneously and quite independently H. Grassmann of Stettin (see above, vol. i. p. 243) had worked out a much more comprehensive and fundamental calculus, of which the method of quaternions and all the different forms of Vector Analysis can be