

received tardy recognition. Such speculations can be carried on either as fascinating exercises of mere ingenuity, or for practical purposes to improve the refined instruments of mathematical calculation, or in the philosophical interest of arriving at the fundamental processes of human thought and intuition.¹ Many persons think that only the second of these three in-

¹ Already Euler had remarked on the different interests that prompted mathematical research. Referring to the writings of Count Fagnano, he says in the introduction to the first of his memoirs on Elliptic Integrals (1761, quoted by Brill & Nöther in 'Bericht der Deutschen Mathematiker-Vereinigung,' vol. iii. p. 206): "If one looks at mathematical speculations from the point of view of utility, they can be divided into two classes: first, those which are of advantage to ordinary life and other sciences, and the value of which is accordingly measured by the amount of that advantage. The other class comprises speculations which, without any direct advantage, are nevertheless valuable because they tend to enlarge the boundaries of analysis and to exercise the powers of the mind. Inasmuch as many researches which promise to be of great use have to be given up owing to the inadequacy of analysis, those speculations are of no little value which promise to extend the province of analysis. Such seems to be the nature of observations which are usually made or found *a posteriori*, but which have little or no chance of being discovered *a priori*. Having once been established as correct, methods more easily present themselves which lead up to them, and there is no doubt that through the search for such methods the domain of analysis may be considerably ex-

tended." The school of mathematicians headed by Abel and Jacobi pursued mathematics from purely scientific interest, and was criticised on this ground by eminent contemporary mathematicians in France: see a letter of Jacobi to Legendre, dated July 2, 1830, in which he refers to a Report of Poisson on his great work, but adds: "M. Poisson n'aurait pas dû reproduire dans son rapport une phrase peu adroite de feu M. Fourier où ce dernier nous fait des reproches, à Abel et à moi, de ne pas nous être occupés de préférence du mouvement de la chaleur. Il est vrai que M. Fourier avait l'opinion que le but principal des mathématiques était l'utilité publique et l'explication des phénomènes naturels; mais un philosophe comme lui aurait dû savoir que le but unique de la science, c'est l'honneur de l'esprit humain et que sous ce titre, une question de nombres vaut autant qu'une question du système du monde." In the sequel he adds: "Je crois entrevoir que toutes ces transcendentes" (*i.e.*, the elliptic and Abelian functions) "jouissent des propriétés admirables et inattendues auxquelles on peut être conduit par le théorème d'Abel. . . . J'ai réfléchi aussi de temps en temps sur une méthode nouvelle de traiter les perturbations célestes, méthode dans laquelle doivent entrer les théories nouvelles des fonctions elliptiques."