

Analytical geometry, by substituting an algebraical expression for a geometrical figure—say a curve,—could apply to it all the artifices of abstract analysis. By varying the co-ordinates you can proceed along the whole extent of the curve and examine its behaviour as it vanishes into infinity, or discover its singular points at which there occurs a break of continuity: you can vary its constants or parameters, and gradually proceed from one curve to another belonging to the same family, as is done in grouping together all curves of the second order, or—as was done in the calculus of variation, invented by Euler and Lagrange—you can vary the form of the equation, proceeding from one class of curve to another. Now clearly all this operating on equations and symbolic expressions was originally abstracted from geometry, including the mechanical conception of motion; in particular the ideas which underlie the method of fluxions were suggested by the motion of a point in space. The conception of continuous motion in space—

the principle as a valuable instrument for the discovery of new truths, which nevertheless did not make stringent proofs superfluous." Cauchy's report seems to have aroused Poncelet's indignation. Hankel (*'Elemente der Projectivischen Geometrie,'* 1875, p. 9) says: "This principle, which was termed by Poncelet the 'Principle of Continuity,' inasmuch as it brings the various concrete cases into connection, could not be geometrically proved, because the imaginary could not be represented. It was rather a present which pure geometry received from analysis, where imaginary quantities behave in all calculations like real ones. Only

the habit of considering real and imaginary quantities as equally legitimate led to that principle which, without analytical geometry, could never have been discovered. Thus pure geometry was compensated for the fact that analysis had for a long time absorbed the exclusive interest of mathematicians; indeed it was perhaps an advantage that geometry, for a time, had to lie fallow." Kötter continues: "Von Staudt was the first who succeeded in subjecting the imaginary elements to the fundamental theorem of projective geometry, thus returning to analytical geometry the present which, in the hands of geometers, had led to the most beautiful results."