By the law of continuity he showed how in pure geometry it became necessary to introduce the consideration of points and lines which vanish into infinity or which become imaginary, establishing by their invisible elements the continuous transition from one geometric form to another; just as in algebra these conceptions had forced themselves on the attention of analysts. Ideal elements were thus made use of to lead to the discovery of real properties.

The consideration of lines and points which vanish or lie at infinity was familiar to students of perspective from the conception of the "vanishing line"; but the inclusion of ideal points and lines was, as Hankel says, a gift which pure geometry received from analysis, where imaginary (*i.e.*, ideal or complex) quantities behave in the same way as real ones. Without the inclusion of these ideal or invisible elements the generality or continuity of purely geometrical reasoning was impossible.

The geometrical reasoning of Monge, Carnot, and Poncelet was thus largely admixed with algebraical or analytic elements. It is true that Monge's descriptive geometry was a purely graphical method, and that

gested to Poncelet by the property, known already to De la Hire ("Sectiones Conicæ," 1685), that in the plane of a conic section every point corresponds to a straight line called its "polar," that to every straight line corresponds a point called its "pole," that the "polars" corresponding to all the points of a straight line meet in one and the same point, and *vice versa* that the "poles" corresponding to all lines going through one and the same point lie on a straight line; the line and point in question standing in both cases in the relation of pole and polar to each other. Poncelet uses "this transformation of one figure into its reciprocal polar systematically as a method for finding new theorems: to every theorem of geometry there corresponds in this way another one which is its 'polar,' and the whole of geometry was thus split up into a series of truths which run parallel and frequently overlap each other" (Hankel, *loc. cit.*, p. 20).