from a point drawing lines or rays on the plane and in space, and we can cut these by lines in a plane or by planes in space. And it can be shown that "if one geometric form has been derived from another by means of one of these operations, we can conversely, by means of the complementary operation, derive the second from the first."[1]

The projective geometry of Poncelet contains the two-fold origin of the principle of duality in his method of projection and section, and in his theory of the reciprocity of certain points and lines in the doctrine of conic sections, called the theory of reciprocal polars. But the mathematician who first expressed the principle of duality in a general—though not in the most general—form was Gergonne, who also recognised that it was not a mere geometrical device but a general philosophical principle, destined to impart to geometrical reasoning a great simplification. He sees in its enunciation the dawn of a new era in geometry.[2]

29. Reciprocity.

[1] Cremona, *loc. cit.*, p. 33.

[2] The principle of Duality seems to have been first put forward in its full generality by Gergonne, inspired probably by the theory of Reciprocal Polars (see note, p. 663) enunciated by Poncelet, who many years afterwards carried on a voluminous polemic as to the priority of the discovery. "Gergonne saw that the parallelism (referred to above) is not an accidental consequence of the property of conic sections, but that it constitutes a fundamental principle which he termed the 'principle of duality.' The geometry which is usually taught, and in which a line is considered to be generated by the motion of a point, is opposed by another geometry equally legitimate in which a point is generated by the rotation of a line. Whereas in the first case the line is the locus of the moving point, in the latter case the point is the geometrical intersection of the rotating line. In this generality the principle of duality has been incorporated into modern geometry" (Hankel, *loc. cit.*, p. 21). Gergonne says of the new principle (1827, see Supplement to vol. ii. 2nd ed. of Poncelet's 'Traité,' p. 390) : "Il ne s'agit pas moins que de commencer pour la géométrie, mal connue depuis près de deux mille ans qu'on s'en occupe, une ère tout-à-fait nouvelle : il s'agit d'en mettre tous les anciens traités à peu près au