The labours of Poncelet and Steiner introduced into geometry a twofold aspect, and accordingly, about the middle of the century, we read a good deal of the two kinds of geometry which for some time seemed to develop independently of each other. The difference has been defined by the terms "analytic or synthetic," "calculative or constructive," "metrical or projective." The one operated with formulæ, the other with figures; the one studied the properties of quantity (size, magnitude), distances, and angles, the other those of position.

The projective method seemed to alter the magnitude of lines and angles and retain only some of those of position and mutual relation, such as contact and intersection. The calculating or algebraical method seemed to isolate figures and hide their properties of mutual interdependence and relation.

31. Mutual influence of metrical and projective geometry.

These apparent defects stimulated the representatives of the two methods to investigate more minutely their hidden causes and to perfect both. The algebraical formula had to be made more pliable, to express more naturally and easily geometrical relations; the geometrical method had to show itself capable of dealing with quantitative problems and of interpreting geometrically those modern notions of the infinite and the complex which the analytic aspect had put promi-

correlated theorem referring to projected ranges), Steiner recognised the fundamental principle out of which the innumerable properties of these remarkable curves follow, as it were, automatically with playful ease. Nothing is wanted but the combination of the simplest theorems and a vivid geometrical imagination capable of looking at the same figure from the most different sides in order to multiply the number of properties of these curves indefinitely" (Hankel, *loc. cit.*, p. 26; see also Cremona, 'Projective Geometry,' p. 119).