

these marvellous works of genius, science is probably indebted for its greatest advances to those mathematicians who, like Plücker in Germany, Chasles in France, and Cayley in England, employed the analytic and constructive methods alternately and with equal mastery.

It is impossible—and it is not my object—to allot to each of these original thinkers the special ideas introduced by him into modern science; but for the purpose

like Johannes Müller could not understand how such simple things could be brought before the Academy of Sciences, whereas the great mathematician Dirichlet was full of praise of the ingenuity of the method by which problems were solved which the Calculus of Variations attacked long after Steiner, and then only in ways which the synthetical method had indicated (see Geiser, 'Zur Erinnerung an Jacob Steiner,' p. 28). It must not be supposed, however, that Steiner was an extreme purist so far as geometrical methods were concerned, for he says himself "that of the two methods neither is entitled to exclude the other; rather both of them will, for a long time, have plenty to do in order to master the subject to some extent, and then only can an opinion as to their respective merits be formed" ('Ges. Werke,' vol. ii. p. 180). An instance of a celebrated problem being treated alternately by synthetic and analytic methods is that of the Attraction of Ellipsoids, in which the Theorem of Maclaurin had created quite a sensation. In spite of the admiration which it evoked, both Legendre and Poisson expressed the opinion that the resources of the synthetic method are easily exhausted. The latter, whilst admitting "que la synthèse ait d'abord devancé l'analyse," never-

theless concludes that "la question n'a été enfin résolue complètement que par des transformations analytiques. . . auxquelles la synthèse n'aurait pu suppléer." This expression of opinion was falsified when Chasles presented to the Academy, in the year 1837, a memoir in which, through the study of confocal surfaces, the Theory of Maclaurin was synthetically proved in its full generality. Poincaré, who reported on this memoir, attached the following remarks: "Ce mémoire remarquable nous offre un nouvel exemple de l'élégance et de la clarté que la géométrie peut répandre sur les questions les plus obscures et les plus difficiles. . . . Il est certain qu'on ne doit négliger ni l'une ni l'autre; elles sont au fond presque toujours unies dans nos ouvrages, et forment ensemble comme l'instrument le plus complet de l'esprit humain. Car notre esprit ne marche guère qu'à l'aide des signes et des images; et quand il cherche à pénétrer pour la première fois dans les questions difficiles, il n'a pas trop de ces deux moyens et de cette force particulière qu'il ne tire souvent que de leur concours. C'est ce que tout le monde peut sentir, et ce qu'on peut reconnaître dans le Mémoire même." (Chasles, 'Rapport sur les progrès de la géométrie,' 1870, p. 105, &c.)