

of bringing some order into the tangled web of mathematical speculation, mainly represented by these, I shall identify the name of Plücker with the great advance which has taken place in geometry through the change in our ideas as to the elements of space construction and the generalisation of our ideas of co-ordinates: with Chasles I shall specially connect the modern habit in geometry of combining figures in finite space with their infinitely distant elements, and with Cayley the application to geometrical science of the novel and comprehensive methods of modern algebra. Let us dwell for a moment on each of these three great departures.

32.  
Plücker,  
Chasles,  
Cayley.

The elements of any science are a very different thing from the elements of the special object with which that science is concerned. The elements of chemistry are not the chemical elements. The latter are, we suppose, something existing in nature, something fixed and unalterable, which science aims at finding out; the former are certain conceptions from which we find it convenient to start in teaching, expounding, and building up the science of chemistry. The latter are artificial, the former are natural. The same remark obtains in geometrical science. The elements of geometry have an historical, a practical beginning: the elements of space form a conception which gradually emerges in the progress of geometrical science. In every science there is a tendency to replace the casual and artificial elements by the natural or real elements, and to build up the historical traditional body of doctrine anew, using the very elements which Nature herself, as it were, employs in producing her actual forms and objects. As the pass-

33.  
Historical  
and logical  
foundations.