cluding arrangement. The same difference of views can be established with regard to many other things which form the objects of other sciences. In geometry this difference obtrudes itself, as it were, in its naked form. Thus in all the natural, and even the social, sciences we have become accustomed to look first at the constituent elements or parts of things, to count and measure them, then afterwards to look at their possible arrangement, or existence together in the actual world of nature or society. Astronomy, crystallography, chemistry, geology, the natural history sciences, economics and statistics, the doctrine of chances,—all furnish, especially in their systematic development during the last hundred or hundred and fifty years, examples of the twofold aspect just referred to. The progress of these sciences, as we have abundantly seen, has depended largely upon the application of mathematical methods. As the analysis into elements or parts, and the possible synthesis of such elements in complicated structures, has become everywhere the order of study, so there must exist in the abstract science of mathematics—*i.e.*, in the framework of our scientific reasoning —not only the theory of measurement and number, but also that of combination, form or arrangement, and order.

87.
Theory of
forms.

The doctrine of forms in the well-known problems of permutations and combinations begins with modern mathematics in the seventeenth century, and received scientific recognition mainly in connection with the doctrine of chances at the hands of James Bernoulli abroad, and of De Moivre in this country. The process of multiplication of binomials and polynomials leads to the formation of combinations, and