

where the factors are the same, as in Newton's binomial theorem, to combinations with permutation; and consequently the doctrine of chances and of arrangements in triangular, pyramidal, or other figures is closely connected with the doctrine of series and algebraical expressions. In this country the interest in the subject has been stimulated and kept alive by isolated problems and puzzles in older popular periodicals, such as the 'Gentleman's Magazine' and the 'Ladies' Diary'; in Germany—as we noticed before—a school of mathematicians arose who attempted a systematic treatment of the whole subject, which, owing to its barrenness in practical results, brought this line of research somewhat into disrepute. What was wanted was a problem of real scientific interest and a method of abbreviation and condensation. Both were supplied from unexpected¹

¹ The theory of arrangement or of order, also called the "Ars Combinatoria," has exerted a great fascination on some master minds, as it has also given endless opportunities for the practical ingenuity of smaller talents; among the former we must count in the first place Leibniz, and in recent times J. J. Sylvester, who conceived the "sole proper business of mathematics to be the development of the three germinal ideas—of which continuity is one, and order and number the other two" ('Philosophical Transactions,' vol. clix. p. 613). This idea has been dwelt on by Major MacMahon in his address (Brit. Assoc., 1901, p. 526), who says: "The combinatorial analysis may be described as occupying an extensive region between the algebras of discontinuous and continuous quantity. It is to a certain extent a science of enumeration, of mea-

surement by means of integers as opposed to measurement of quantities which vary by infinitesimal increments. It is also concerned with arrangements in which differences of quality and relative position in one, two, or three dimensions are factors. Its chief problem is the formation of connecting roads between the sciences of discontinuous and continuous quantity. To enable, on the one hand, the treatment of quantities which vary *per saltum*, either in magnitude or position, by the methods of the science of continuously varying quantity and position, and, on the other hand, to reduce problems of continuity to the resources available for the management of discontinuity. These two roads of research should be regarded as penetrating deeply into the domains which they connect."