

foremost in having with unrivalled fertility propounded theorems which were as difficult to prove as the manner in which they had been arrived at was mysterious. The great analytical genius of Euler, who possessed unequalled resources in the solution of single problems, spent much time and power in unravelling the riddles of Fermat. In the theory of equations the general solution beyond the fourth degree baffled the greatest thinkers. The time had come when in both branches a systematic study of the properties had to be attempted. This was done for the theory of numbers by Gauss, for that of equations by Abel. Every great step in advance of this kind in mathematics is accompanied by, and dependent on, skilful abbreviations, and an easy algorithm or mathematical language. An assemblage of elements held together by the simplest operations or signs of arithmetic—namely, those of addition and multiplication—is much easier to deal with if it can be arranged with some regularity, and accordingly methods were invented by which algebraical expressions or forms were made symmetrical and homogeneous;¹ the latter property signifying that each term

39.
Symmetry.

Arithmetic,' gave an interesting theorem by which the number of imaginary roots of an equation can be determined; he left no proof, and the theorem was discussed by Euler and many other writers, till at last Sylvester in 1866 found the proof of it in a more general theorem. In more recent times Jacob Steiner published a great number of theorems referring to algebraical curves (see Crelle's 'Journal,' vol. xlvii.) which have been compared by Hesse with the "riddles of Fermat." Luigi Cremona succeeded at last in proving

them by a general synthetical method.

¹ The introduction of homogeneous expressions marks a great formal advance in algebra and analytical geometry. The first instance of homogeneous co-ordinates is to be found in Möbius's "Barycentric Calculus" (1826), in which he defined the position of any point in a plane by reference to three fundamental points, considering each point as the centre of gravity of those points when weighted. "The idea of co-ordinates appears here for the first time in a new