

contained the same number of factors. Such forms could be written down on the pattern or model of one of their terms by simple methods of exchange or permutation of the elements. It would then not be necessary to write down all the terms but only to indicate them by their elements, these also being abbreviated by the use of indices. Rows and columns or arrangements in squares suggested themselves as easy and otherwise well-known artifices by which great masses of statistics and figures are marshalled and controlled. Out of these manifold but simple devices there grew an algebra of algebra, a symbol for denoting in a very general way symmetrical and homogeneous algebraical expressions.<sup>1</sup> Gauss termed such expressions Determinants: they turned up in his 'Disquisitiones Arithmeticae' as they had done half a century before in Cramer's 'Analyse des lignes courbes algébriques.' Just as common fractions can be

40.  
Determinants.

garb, which soon led to a more general conception. The Barycentric co-ordinates were the first instance of homogeneous co-ordinates, . . . and already with Möbius the advantages become evident through the symmetry and elegance of his formulæ" (Hankel, 'Project. Geom.,' p. 22).

<sup>1</sup> Determinants were first used by Leibniz for the purpose of elimination, and described by him in a letter to the Marquis de l'Hospital (1693). The importance of his remarks was not recognised and the matter was forgotten, to be rediscovered by Cramer in the above-named work (1750, p. 657). It is interesting to note that the same difficulty of the process of elimination induced Plücker to resort to geometrical

interpretation of analytical expressions, and that whilst he "saw the main advantage of his method in avoiding algebraical elimination through a geometrical consideration, Hesse showed how, through the use of Determinants, algebraical operations could receive that pliability the absence of which was the reason for Plücker to discard it." (See the account of Clebsch's work in 'Math. Ann.,' vol. vii. p. 13.) Through this invention the combinatorial analysis, which, in the hands of the school in Germany, had led into a desert, was raised again into importance. It has become still more important since the general theory of forms and of groups began to play an increasing part in modern analysis.