dealt with as if they were special things having special properties, though the latter depend only on the properties of the numbers they are made up of and their mode of connection; as powers and surds are separately examined; so the arrangements called determinants can be subjected to a special treatment, their properties ascertained, and themselves subjected to the ordinary operations of arithmetic. This doctrine, which constitutes the beginning and centre of the theory of algebraical forms or "quantics" and of algebraical operations or "tactics," was pretty fully worked out and first introduced into the course of teaching by Cauchy in France; then largely adopted by Jacobi in Germany, where Otto Hesse, trained in the ideas of Plücker, first showed its usefulness in his elegant applications to geometry. In France it was further developed by Hermite, who, together with Cayley and Sylvester in England, proclaimed the great importance of it as an instrument and as a line of mathematical thought.[1] In the latter country the idea of abbreviating and summarising algebraical operations had become quite familiar through another device which has not found equal favour abroad—namely, the Calculus of

[1] "For what is the theory of determinants? It is an algebra upon algebra; a calculus which enables us to combine and foretell the results of algebraical operations, in the same way as algebra enables us to dispense with the performance of the special operations of arithmetic. All analysis must ultimately clothe itself under this form." In this connection Sylvester ('Phil. Mag.,' 1851, Apl., p. 301) refers to Otto Hesse's "problem of reducing a cubic function of three letters to another consisting only of four terms by linear substitutions—a problem which appears to set at defiance all the processes and artifices of common algebra," as "perhaps the most remarkable indirect question to which the method of determinants has been hitherto applied."