41. Calculus of Operations.

Operations, the idea of treating algebraical operations and their symbols as quantities, and of subjecting them to arithmetical treatment separately from the material operated on. The genius of Arthur Cayley was specially fertile in this direction, as was that of Sylvester in the nomenclature or language of the doctrine of forms.[1] The merit, however, of having brought together the new ideas which emanated from the schools of Poncelet and Chasles in France, of Cayley and Sylvester in England, into a connected doctrine, and of having given the impetus to the fundamental re-

[1] The theory of invariants was gradually evolved from many independent beginnings. In 1864 Sylvester wrote ('Phil. Trans.,' p. 579), "As all roads are said to lead to Rome, so I find, in my own case at least, that all algebraical inquiries, sooner or later, end at the Capitol of Modern Algebra, over whose shining portal is inscribed the Theory of Invariants." About the same time (1863) Aronhold developed the principal ideas which lay at the foundation of the theory in organic connection and in complete generality, hereby domiciling in Germany the doctrine which had previously owed its development mainly to English, French, and Italian mathematicians (see Meyer, 'Bericht,' &c., p. 95). The different roads which Sylvester refers to can be traced, first, in the love of symbolic reasoning of Boole, who was "one of the most eminent of those who perceived that the symbols of operation could be separated from those of quantity and treated as distinct objects of calculation, his principal characteristic being perfect confidence in any result obtained by the treatment of symbols in accordance with their primary laws and conditions, and an almost unrivalled skill and power in tracing out these results" (Stanley Jevons in article "Boole," 'Ency. Brit.'); secondly, in the independent geometrical labours of Hesse in Germany (whose mathematical training combined Plücker's and Jacobi's teaching) and Dr Salmon in Dublin (who, after having transplanted Poncelet and Chasles to British soil, recognised the importance of Cayley's and Sylvester's work, and introduced in the later editions of his text-book modern algebraical methods); thirdly, in the independent investigations belonging to the theory of numbers of Eisenstein in Germany and Hermite in France. In full generality the subject was taken up and worked out by Sylvester in the 'Cambridge and Dublin Mathematical Journal' (1851-54), and by Cayley in the first seven memoirs upon Quantics (1854-61), which "in their many-sidedness, together with the exhaustive treatment of single cases, remain to the present day, for the algebraist as well as for the geometrician, a rich source of discovery" (Meyer, *loc. cit.*, p. 90).