42. Principle of substitution.

counterpart in the transformation of algebraical forms by the processes of substitution, these latter had already been extensively studied for their own sakes in the theory of algebraical equations, which in the first quarter of the century had undergone a great development under the hands of two brilliant mathematical talents both lost to science at an early age—the Norwegian Abel and the Frenchman Évariste Galois.[1]

Like all algebraical expressions, those termed equations were originally invented and commanded attention

[1] Évariste Galois is held to have been one of the greatest mathematical geniuses of modern times, who, if he had lived, might have been a rival of Abel: he was born in 1811, and died before he was twenty-one, in consequence of a duel. For a long time his writings remained unpublished and unknown, till Liouville published them in the 11th vol. of his 'Journal' (1846). Liouville was also the first to recognise the importance and absolute correctness of Galois's method, which, when submitted to the Academy in the year 1831, and reported on by Lacroix and Poisson, had appeared almost unintelligible. On the eve of his death Galois addressed a letter to his friend Auguste Chevalier, which is a unique document in mathematical literature, forming a kind of mathematical testament. He desires this letter to be published in the 'Revue Encyclopédique,' referring publicly the "importance," not the "correctness," of his discoveries to the judgment of Jacobi and Gauss, and expressing the hope that some persons would be found who would take the trouble to unravel his hieroglyphics. The first attempt to make Galois's ideas generally accessible is to be found in Serret's 'Algèbre Supérieure' (3rd ed., 1866), but it was not till after the publication of Camille Jordan's 'Théorie des Substitutions' (1870) that the short papers of Galois were recognised as containing the germs and beginnings of an entirely novel and comprehensive mathematical theory—viz., the "Theory of Groups." The relation between the writings of Abel and Galois is exhaustively treated in Prof. Sylow's Paper on Abel's work, contained in the 'Memorial Volume,' 1892, p. 24. He there says: "Le mérite de Galois ne consiste pas essentiellement dans ses propositions, mais dans la généralité de la méthode qu'il appliqua. C'est son admirable théorème fondamental qui a donné à la théorie des équations sa forme définitive, et d'où est sortie, en outre, la théorie des groupes généralisée, qui est d'une si grande importance, on peut le dire, pour toutes les branches des mathématiques, et qui déjà, entre les mains de Jordan, de Klein, de Lie, de Poincaré et d'autres, a enrichi la science d'une longue suite de découvertes importantes." The memoirs of Abel and Galois referring to the Theory of Equations have been conveniently edited, in a German translation, by H. Maser, 1889. See also Cayley's article on "Equation" in the 'Ency. Brit.,' § 32.