as instruments or devices for the solution of definite problems in arithmetic, geometry, and mechanics. The solution of the equation—*i.e.*, the expression of the unknown quantity in terms of the known quantities—served a practical end. Gradually as such solutions became more and more difficult, owing to the complexity of the formulæ, the doctrine divided itself into two distinct branches, serving two distinct interests. The first, and practically the more important one, was to devise methods by which in every single case the equations which presented themselves could be solved with sufficient accuracy or approximation; this is the doctrine of the numerical solution of equations. The other more scientific branch looked upon equations as algebraical arrangements of quantities and operations which possessed definite properties, and proposed to investigate these properties for their own sake. The question arose, How many solutions or roots an equation would admit of, and whether the expression of the unknown quantity in terms of the known quantities was or was not possible by using merely such operations as were indicated by the equation itself—*i.e.*, the common operations and the ordinary numbers of arithmetic? This doctrine of the general properties of equations received increasing attention as it became empirically known that equations beyond the fourth degree could not be solved in the most general form.[1] Why could they not be solved,

43. General solution of equations.

[1] Since the researches regarding the solubility of Equations have led on, through Galois and the French analysts, to the same line of reasoning as other researches mentioned before—viz., toward the development of the theory of groups—the history of the whole subject has aroused special interest. The earlier beginnings and the labours of forgotten analysts have been un-