

and what were the conditions—*i.e.*, the special properties—of an equation which rendered it soluble? These were some of the questions which the great mathematicians, such as Gauss, Abel, and Galois, placed before themselves during the earlier part of the century. There are other unsolved problems which the nineteenth century inherited from preceding ones, where the same line of reasoning was adopted—*i.e.*, where the question was similarly reversed. Instead of trying to solve problems as yet unsolved, it was proposed to prove their general insolubility, and to show the reason of this; also to define the conditions which make a solution possible.

earthed and placed in their correct historical perspective. Prof. Burkhardt of Göttingen, to whom we also owe the chapter on this subject in the first volume of the 'Encyklopädie,' &c., contributed in the year 1892 a most interesting historical paper, "Die Anfänge der Gruppentheorie und Paolo Ruffini" ('Abhandl. zur Gesch. der Math.,' 6 Heft). In this paper he also goes back to other earlier analysts, among them Prof. Waring of Cambridge, who during his lifetime used to complain that he knew of no one who read his mathematical tracts. It appears that during nearly the last thirty years of the eighteenth century nothing had been added regarding the general theory of equations, and that Ruffini was the first to begin a new epoch in the year 1799, with the distinct assertion that a general solution of algebraic equations beyond the fourth degree, by means of radicals, was impossible, and with an attempt to prove this. His researches were therefore contemporaneous with those of Gauss, who published his 'Dissertation' (see note p. 644) in the same year, and his great arithmetical work

in 1801. Although Gauss seems to have arrived at the same conclusion, and perhaps even to have anticipated much later attempts to solve the general equation of the fifth degree by other than algebraical operations (see Sylow, *loc. cit.*, p. 16), his published researches rather took the line of the study of a definite class of soluble equations which were connected with the celebrated problem of the division of the circle; a satisfactory proof of Ruffini's statement being withheld till Abel published his celebrated memoir in the year 1825 in the first volume of Crelle's 'Journal.' With this memoir the theory of equations entered a new phase, towards which the labours of Ruffini were preparatory. As in so many other cases, so also in this, the solution of the problem depended upon stricter definitions of what was meant by the solution of an equation, and by "algebraical" and other ("transcendental") functions and operations. We know that both Abel and Galois began their research by futile attempts to find a solution of the general equation of the fifth degree.