In following this altered course of investigation, an enormous amount of mathematical knowledge was gained, and problems were solved which had previously never been thought of. Especially through the theory of equations the abstract doctrine of algebraical forms was created and greatly advanced long before it was generally recognised that it had peculiar importance through the correspondence or parallelism which existed between algebraical expressions and geometrical configurations.

Out of these earlier algebraical and later combined algebraical and geometrical investigations, a novel and very useful point of view has been gradually gained which represents the most general conception of mathematical tactics. This centres in the notion of a group of elements. These elements may be quantities or operations, so that the theory of Groups embraces not only the doctrines which deal with quantities but also those which deal with arrangements and their possible changes. The older combinatorial analysis dealt mainly with assemblages of a quantity of separate elements, their number, their variety: the modern theory of groups deals rather with the processes and operations by which different arrangements can be transformed one into the other. It is an algebra of operations. The methods of transformation which presented themselves first of all were the methods known in algebra as substitution. Accordingly the first comprehensive treatise on the theory was the 'Treatise on Substitutions,' published in 1870 by M. Camille Jordan. This book forms a landmark in modern mathematics; it brought into a system