the beginnings of the new and comprehensive calculus of operations which were contained in the writings of Lagrange, Abel, Cauchy, and Galois, and established the terminology and the algorithm. A group of substitutions is defined as having the property that each two or more operations belonging to it and successively applied can be replaced by another single operation contained in the same group. Succeeding operations are symbolically represented by the product of two or more letters. This product has certain algebraical properties, and in analogy with common products it has factors, a degree, an index; the substitution may be cyclical and symmetric, and may have many other remarkable properties which the theory[1]

[1] The "Theory of Groups" has now grown into a very extensive doctrine which, according to the late Prof. Marius Sophus Lie (1842-99), is destined to occupy a leading and central position in the mathematical science of the future. "The conception of Group and Invariant was for him not only a methodical aspect from which he intended to review the entire older region of mathematics, but also the element which was destined to permeate and unify the whole of mathematical science" (M. Nöther, 'Math. Ann.,' vol. liii. p. 39). But though it is an undoubted fact that the largest systematic works on the subject emanate from that great Norwegian mathematician, and that his ideas have won gradual recognition, especially on the part of prominent French mathematicians, notably M. Picard ('Traité d'Analyse,' 1896, vol. iii.) and M. Poincaré, the epoch-making tract which pushed the novel conception into the foreground was Prof. F. Klein's 'Erlangen Programme' (1872), entitled "Vergleichende Betrachtungen über neuere geometrische Forschungen." To those who read and re-read this short but weighty treatise, it must indeed have been like a revelation, opening out entirely new avenues of thought into which mathematical research has been more and more guided during the last generation. The tract, which has now been translated into all the important modern languages, remained for a long time comparatively unnoticed, and, twenty years after its publication, was reprinted by the author in the 43rd volume of the 'Math. Annalen,' with some introductory remarks which indicate the changes that had taken place in the interval as regards the scope of the idea. The main result of the dissertation is this: That, primarily, for all geometrical investigations, the characteristic properties of any manifold (or arrangement) is not the element out of which it is composed, but the group, the transformations of which reveal its invariantive properties. There are, accordingly, as many different ways of