of groups investigates. Its immediate application, and the purpose for which it was elaborated, was the theory of Equations. Every equation constitutes an arrangement in which a finite number of independent elements, called constants or coefficients, is presented under a certain algebraical form. The solution of the equation means the finding of such an arrangement as when substituted in the equation for the unknown quantity, will satisfy the equation.

The conception of a group of operations standing in the defined relations is, however, capable of a great and fundamental extension into that region of mathematics which deals, not with fixed or constant, but with variable or flowing quantities; not with elements which are disconnected or discontinuous, but with such as are continuous. To understand the development of modern mathematical thought, it is accordingly necessary to go back somewhat and review the progress which the

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studying any manifold (*e.g.*, such as projective geometry, line geometry, geometry of reciprocal radii, Lie's sphere geometry, analysis situs, &c.) as there are continuous groups of transformations that can be established ; and there are as many invariant theories (see 'Ency. Math. Wiss.,' vol. ii. p. 402; Nüther, *loc. cit.*, p. 22). From that date onward the different kinds of groups have been defined and systematically studied, notably by Klein and Lie and their pupils. In this country, although many of the relevant ideas were contained in the writings notably of Cayley and of Sylvester, the systematic treatment of the subject was little attended to before the publication (1897) of Prof. Burn-side's 'Theory of Groups of Finite Order,' and latterly of his article on the whole Theory of Groups in the 29th volume of the 'Ency. Brit.' It has been remarked by those who have studied most profoundly the development of the two great branches of mathematical tactics —viz., "The Theory of Invariants" and the "Theory of Groups"—that the progress of science would have been more rapid if the English school had taken more notice of the general comprehensive treatment by Lie, and if Lie himself had not refrained from entering more fully into the special theories of that school (see Dr F. Meyer, 'Bericht,' &c., p. 231).