other.   The question arises, What are we to under-
stand under this term?   What is a mathematical
function or dependence?   The question was approached
by the great analysts of the second half of the
eighteenth century.   A preliminary answer which served
the requirements of a very wide field of practical
application was given by Fourier at the beginning of
the nineteenth century.   Since that time the question
has been independently treated by two schools of
Continental mathematicians.   Of these the first was
founded by Cauchy in France, and is mainly represented
by Bernhard Riemann and his numerous pupils in
Germany; the other centres in the Berlin school,
headed by Weierstrass, and goes back to the work
of Lagrange.

The interests which have led to this modern branch
of mathematical research [1] are various, but we can

46.
Theory of
Functions.

The literature suitable for intro-
ducing the student of mathematics
to the modern theory of functions
—which plays in analysis, i.e., the
doctrine of variable quantity, a
part of similar importance to that
which the theory of forms plays in
algebra—is so enormous, the sub-
ject being approached from so
many sides by different writers,
that it seems worth while to refer
to two expositions which may be
read with profit, and which do
not require extensive mathematical
knowledge.   First and foremost I
would recommend Cayley's article
on "Functions" in vol ix. of the
'Ency. Brit.'   Then there is the
chapter on "Foundations of the
General Theory of Functions," con-
tained in the 2nd volume of the
German 'Mathematical Encyclo-
pedia,' written by Prof. Prings-

heim.   Cayley's article introduces
the general theory after giving
a short summary of the more
important "known" functions,
including those which presented
themselves in the first half of
the nineteenth century, and which
I referred to in dealing with
the work of Abel and Gauss (see
note, p. 648).   The treatment of
these latter functions, which had
been brought to a certain degree of
perfection by Jacobi, had made it
evident that more general aspects
had to be gained and broader
foundations laid.   But ever since
the middle of the eighteenth
century another development of
mathematical ideas had been going
on which started from the solu-
tion of a problem in mathematical
physics—namely, that of vibrating
strings, which led in the sequel to