

distinguish two which are very prominent, and are roughly represented by the two schools just referred to. In the first place, a function can be formally defined as an assemblage of mathematical symbols, each of which denotes a definite operation on one or more quantities. These operations are partly direct, like addition, multiplication, &c.; partly indirect or inverse, like subtraction, division, &c. Now, so far as the latter are concerned, they are not generally and necessarily practicable, and the question arises, When are they practicable, and if they are not, what meaning can we connect with the mathematical symbol? In this way we arrive at definitions for mathematical functions which cannot immediately be reduced to the primary operations of arithmetic, but which form special expressions that become objects of research as to their properties and as to the relation they bear to those fundamental operations upon which all our methods of calculation depend. The inverse operations, represented by negative, irrational, and imaginary quantities; further, the operations of integration in its definition as the

a certain finality when Fourier introduced his well-known series and integrals, by which any kind of functionality or mathematical dependence, such as physical processes seem to indicate, could be expressed. The work of Fourier, which thus gave, as it were, a sort of preliminary specification under which a large number of problems in physical mathematics could be attacked and practically solved, together with the stricter definitions introduced by Lejeune Dirichlet, settled for a time and for practical purposes the lengthy discussions which had begun with

Euler, Daniel Bernoulli, d'Alembert, and Lagrange. The above-named chapter, written by Prof. Pringsheim, gives an introduction to the subject showing the historical genesis of the conception of function and the various changes it was subjected to, and then proceeds to expositions and definitions mostly taken from the lectures of Weierstrass (see p. 8), whereas Cayley's article introduces us to the elements of the general theory of functions as they were first laid down by Riemann in the manner now commonly accepted.