direction of promoting these seemingly abstract researches. Nature herself exhibits to us measurable and observable quantities in definite mathematical de-

47. Physical analogies.

pendence;[1] the conception of a function is suggested by all the processes of nature where we observe natural phenomena varying according to distance or to time.

[1] Nearly all the "known" functions have presented themselves in the attempt to solve geometrical, mechanical, or physical problems, such as finding the length of the arc of the ellipse (elliptic functions); or answering questions in the theory of attraction (the potential function and other functions, such as the functions of Legendre, Laplace, and Bessel, all comprised under the general term of "harmonic functions"). These functions, being of special importance in mathematical physics, were treated independently before a general theory of functions was thought of. Many important properties were established, and methods for the numerical evaluation were devised. In the course of these researches other functions occurred, such as Euler's "Gamma" function and Jacobi's "Theta" function, which possessed interesting analytical properties. These functions, suggested directly or indirectly by applications of analysis, did not always present themselves in a form which indicated definite analytical processes, such as processes of integration or the summation of series. Very frequently they presented themselves, not in an "explicit" but in an "implicit" form; their properties being expressed by certain conditions which they had to fulfil. It then remained a question whether a definite symbol, indicating a set of analytical operations, could be found. This arises from the fact that the solution of most problems in mechanics and physics starts from the assumption that, though the finite observable phenomena of nature are extremely intricate, they are, nevertheless, compounded out of comparatively simple elementary processes, which take place between the discrete atoms, or the elementary but continuous portions of matter. Mathematically expressed, this means that the relations in question present themselves in the form of differential equations, and that the solution of them consists in finding functions of finite (observable) quantities which satisfy the special conditions. A comparatively small number of differential equations has thus been found empirically to embrace very large and apparently widely separated classes of physical phenomena, suggesting physical relations between those phenomena which might otherwise have remained unnoticed. The physicist or astronomer thus hands over his problems to the mathematician, who has either to integrate the differential equations, or, where this is not possible, at least to infer the properties of the functions which would satisfy them —in fact, the differential equation becomes a definition of the function or mathematical relation. In consequence of this the theory of differential equations is, as Sophus Lie has said, by far the most important branch of mathematics.