The attraction of the heavenly bodies varies with the distance, the velocity of a falling stone or the cooling of a hot body varies with the interval of time which has lapsed or flown. We are now so much accustomed to represent such dependence by curves drawn on paper, that we hardly realise the great step in advance towards definiteness and intelligibility that this device marks in all natural sciences and in many practical pursuits. But the representation of the natural connections of varying quantities by curves also forms the connecting link with the other class of researches just mentioned. Descartes had shown how to represent algebraical formulæ by curves in the plane and in space; and at the beginning of the nineteenth century this method was modified by Gauss and Cauchy so as to deal also with the extended conception of number which embraced the imaginary unit. Two questions arise, Is it possible to represent every arbitrary dependence such as we meet with in the graphical description of natural phenomena by a mathematical formula—*i.e.*, by a formula denoting several specified mathematical operations in well-defined connections? and the inverse question, Is it possible to represent every well-defined arrangement of symbols denoting special mathematical operations graphically by curves in the plane or in space? The former question is one of vital importance in the progress of astronomy, physics, chemistry, and many other sciences, and has accordingly occupied many eminent analysts ever since Fourier gave the first approximative answer in his well-known series: the latter question can only be answered by much stricter defini-