tions of all the more advanced and of some even of the elementary operations which analysts had become accustomed to use without a previous knowledge of the range of their validity. All applications of mathematics consist in extending the empiricial knowledge which we possess of a limited number or region of accessible phenomena into the region of the unknown and inaccessible; and much of the progress of pure analysis consists in inventing definite conceptions, marked by symbols, of complicated operations; in ascertaining their properties as independent objects of research; and in extending their meaning beyond the limits they were originally invented for,—thus opening out new and larger regions of thought.

48. The potential.

A brilliant and most suggestive example of this kind of reasoning was afforded by a novel mode of treating a large class of physical problems by means of the introduction of a special mathematical function, termed by George Green, and later by Gauss, the "Potential" or "Potential function."[1] All the problems of Newtonian attraction were concentrated in the study of this formula: and when the experiments of Coulomb and Ampère showed the analogy that existed between electric and magnetic forces on the

[1]. See vol. i. p. 231 of this work. The history of the subject has been written by Todhunter ('History of the Theories of Attraction and the Figure of the Earth,' 2 vols., 1873) for the earlier period down to 1832. For the later period see Bacharach's 'Abriss der Geschichte der Potentialtheorie,' Göttingen, 1883; for the connection of the theory with Riemann's mathematical methods, especially Prof. F. Klein's tract, 'Ueber Riemann's Theorie der algebraischen Functionen' (Leipzig, 1882, trans. by F. Hardcastle, Cambridge, 1893); Prof. Carl Neumann's 'Untersuchungen über das Logarithmische und Newtonische Potential' (Leipzig, 1877); Dr Burkhardt's 'Memorial Lecture on Riemann' (Göttingen, 1892); and jointly with Dr Franz Meyer, the same author's chapter on "Potentialtheorie" in the 2nd volume (p. 464) of the 'Encyclopädie der Math. Wiss.,' 1900.