49. Riemann.

The peculiarity of such dependence, as exemplified in the phenomena of the steady flow of heat or of electric distribution, consisted in this, that if at certain points or in certain regions of space the thermal or electrical conditions were defined and known by actual observation, then the whole distribution in other points and regions was completely determined. Those boundary conditions could therefore be regarded as the necessary and sufficient definition of the whole existing distribution. Translated into mathematical language, this means that functions exist which are completely defined by boundary values and singularities—*i.e.*, values at single points. Nature herself had shown the way to define and calculate measured relations when through their intricacy they evaded the grasp of the ordinary operations of algebra.[1] Plücker had already in geometry (following in the lines of Newton), when attacking the problem of the infinite variety of higher curves, suggested the method of classifying them according to their characteristic properties or singularities. What had been done by geometers and physicists in isolated cases with the expenditure of much ingenuity and skill, Riemann and his school elevated to the rank of a general method and doctrine.

functions acquires a great degree of clearness and connectedness, which is mainly gained by conceptions derived from the (physical) theory of the potential, and thus exhibits the intimate relationship of these theories" (Bacharach, 'Geschichte der Potentialtheorie,' Göttingen, 1883, p. 71).

[1] On this subject see Burkhardt's 'Memorial Lecture on Riemann' (Göttingen, 1892), p. 5, &c.; Bacharach (*loc. cit.*), p. 30, &c. The latter especially with reference to the theorem called by Clerk-Maxwell "Thomson's theorem" ('Cambridge and Dublin Mathematical Journal,' 1848, or 'Reprint of Papers on Electro-statics,' &c., p. 139); and abroad 'Dirichlet's Principle,' after Riemann (1857). Further, Brill and Nöther's "Bericht" ('Math. Ver.,' vol. iii. p. 247); and lastly, a very suggestive address by Prof. Klein ("On Riemann's Influence on Modern Mathematics") to the meeting of the German Association in Vienna in 1804 ('Report,' p. 61).