It is a process of generalisation and simplification. Moreover, Riemann's manner of proceeding brought with it the gain that he could at once make the various theorems of the doctrine of the potential useful for purely mathematical purposes: the equation which defined the potential in physics became the definition of a function in mathematics.[1]

[1] "One may define Riemann's developments briefly thus: that, beginning with certain differential equations which the functions of the complex variable satisfy, he is enabled to apply the principles of the potential theory. His starting-point, accordingly, lies in the province of mathematical physics" (Klein, 'Vienna Report,' *loc. cit.*, p. 60). By starting with physical analogies Prof. Klein evades certain difficulties which the purely mathematical treatment had to encounter. In the preface to his tract of the year 1882, quoted above,—in introducing his method of explaining Riemann's theory,—he says: "I have not hesitated to make exactly these physical conceptions the starting-point of my exposition. Instead of them, Riemannn, as is well known, makes use in his writings of Dirichlet's principle. But I cannot doubt that he started from those physical problems, and only afterwards substituted Dirichlet's principle in order to support the physical evidence by mathematical reasoning. Whoever understands clearly the surroundings among which Riemann worked at Göttingen, whoever follows up Riemann's speculations as they have been handed down to us, partly in fragments, will, I think, share my opinion." And elsewhere he says: "We regard as a specific performance of Riemann in this connection the tendency to give to the theory of the potential a fundamental importance for the whole of mathematics, and further a series of geometrical constructions or, as I would rather say, of geometrical inventions" ('Vienna Report,' p. 61). Klein then refers to the representation on the so-called "Riemann surface," which is historically connected, as Riemann himself points out, with the problem which Gauss first attacked in a general way—viz., the representation of one surface on another in such a manner that the smallest portions of the one surface are similar to those of the other: a problem which is of importance in the drawing of maps, and of which we possess two well-known examples in the stereographic projection of Ptolemy and the projection of Mercator. This method of representation was called by Gauss the "Conformal Image or Representation." His investigations on this matter were suggested by the Geodetic Survey of the kingdom of Hanover, with which he was occupied during the years 1818 to 1830. (See Gauss, 'Werke,' vol. iv., also his correspondence with Schumacher and Bessel.) A very complete treatise on this aspect of Riemann's inventions is that by Dr J. Holtzmüller, 'Theorie der Isogonalen Verwandschaften' (Leipzig, 1882). On the historical antecedents of Riemann's conception, which for a long time appeared somewhat strange, not to say artificial, see Brill and Nöther's frequently quoted "Report" ('Bericht der Math. Verein.,' vol. iii.), p. 256 *sqq*.